14A

(1) (a) $(4,6)$ $r=5$

$$(x-a)^2+(y-b)^2=r^2$$
$$(x-4)^2+(y-6)^2=5^2$$
$$(x-4)^2+(y-6)^2=25$$

(d) $(-3,-5)$ 6

$$(x--3)^2+(y--5)^2=6^2$$
$$(x+3)^2+(y+5)^2=36$$

(g) $(x-0)^2+(y-0)^2=(\sqrt{5})^2$

$$x^2+y^2=5$$

(2) $(x-a)^2+(y-b)^2=r^2$

(b) $(x+4)^2+(y-1)^2=9$ $\quad a=-4,\ b=1,\ r^2=9, r=3$

$C(-4,1),\ r=3$

(e) $(x+3)^2+y^2=84$ $\quad a=-3\quad b=0\quad r^2=84, r=\sqrt{84}$
$=\sqrt{4}\sqrt{21}$
$=2\sqrt{21}$

$C(-3,0), r=2\sqrt{21}$

(h) $(x+4)^2+(y-5)^2=7$ $\quad a=-4\quad b=5\quad r^2=7\quad r=\sqrt{7}$

$C(-4,5), r=\sqrt{7}$

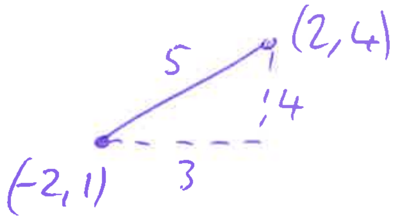
14A

(4) $C(0,7)$ $D=10$ so $r=5$

$$(x-0)^2+(y-7)^2=5^2$$

$$\underline{\underline{x^2+(y-7)^2=25}}$$

(5) $C(-2,1)$, $r=5$

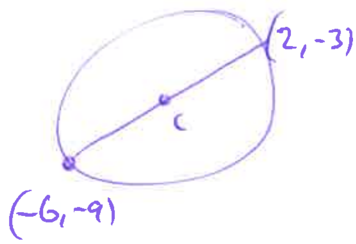


$$(x--2)^2+(y-1)^2=5^2$$

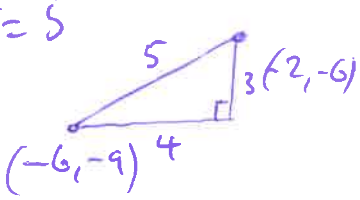
$$\underline{\underline{(x+2)^2+(y-1)^2=25}}$$

(7)

	C	
-6	-2	2
-9	-6	-3



$C(-2,-6)$, $r=5$



$$(x--2)^2+(y--6)^2=5^2$$

$$\underline{\underline{(x+2)^2+(y+6)^2=25}}$$

14A

8 $C(4,a)$

$$(x-4)^2+(y-a)^2=r^2$$

sub in $(-1,3)$

$$(-1-4)^2+(3-a)^2=r^2$$
$$(-5)^2+(3-a)^2=r^2$$
$$25+(3-a)^2=r^2 \quad (1)$$

sub in $(1,1)$

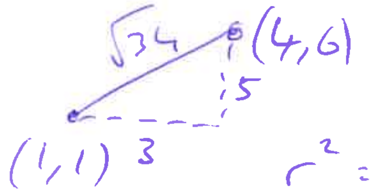
$$(1-4)^2+(1-a)^2=r^2$$
$$(-3)^2+(1-a)^2=r^2$$
$$9+(1-a)^2=r^2 \quad (2)$$

$$16+(3-a)^2-(1-a)^2=0 \qquad (1)-(2) \text{ eliminates } r^2$$

$$16+(3-a)(3-a)-(1-a)(1-a)=0$$
$$16+(9-6a+a^2)-(1-2a+a^2)=0$$
$$16+9-6a+a^2-1+2a-a^2=0$$
$$24-4a=0$$
$$24=4a$$
$$a=\frac{24}{4}$$
$$\underline{\underline{a=6}} \qquad \text{so } \underline{\underline{C(4,6)}}$$

14A

(8) continued



$r^2 = 3^2 + 5^2$

$r^2 = 34$

$r = \sqrt{34}$

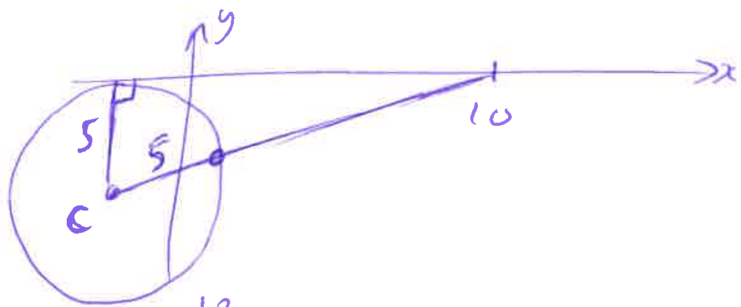
$(x-a)^2 + (y-b)^2 = r^2$

$(x-4)^2 + (y-6)^2 = 34$

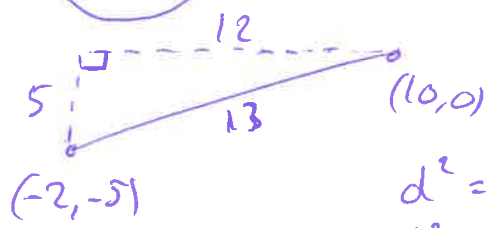
(9) $(x+2)^2 + (y+5)^2 = 25$

$(x-a)^2 + (y-b)^2 = r^2$

$C(-2,-5)$, $r = 5$



Distance $C(-2,-5)$ to $(10,0)$



$d^2 = 12^2 + 5^2$

$d^2 = 169$

$d = 13$

Shortest distance $= 13 - \text{radius}$

$= 13 - 5$

$= 8$ units

14B

For a circle to exist $g^2+f^2-c>0$

① (a) $x^2+y^2+4x+4y+3=0$

$x^2+y^2+2gx+2fy+c=0$

$2g=4 \qquad 2f=4 \qquad c=3$

$g=2 \qquad f=2$

g^2+f^2-c

$=2^2+2^2-3$

$=5. >0$

As $g^2+f^2-c>0$ the equation represents a circle.

(e) $x^2+y^2-6y-2=0$

$x^2+y^2+2gx+2fy+c=0$

$2g=0 \qquad 2f=-6 \qquad c=-2$

$g=0 \qquad f=-3$

g^2+f^2-c

$=0^2+(-3)^2-(-2)$

$=11>0$

$g^2+f^2-c>0$ the equation represents a circle.

14B

2(b) $x^2 + y^2 - 6x + 4y + 4 = 0$

$x^2 + y^2 + 2gx + 2fy + c = 0$

$2g = -6 \quad 2f = 4 \quad c = 4$

$g = -3 \quad f = 2$

$C(-g, -f)$

$C(3, -2)$

radius $= \sqrt{g^2 + f^2 - c}$

$= \sqrt{(-3)^2 + (2)^2 - 4}$

$= \sqrt{9}$

$= 3$

$C(3, -2), \ r = 3$

2(d) $x^2 + y^2 - 6x - 10y + 17 = 0$

$2g = -6 \quad 2f = -10 \quad c = 17$

$g = -3 \quad f = -5$

$C(-g, -f)$

$C(3, 5)$

$r = \sqrt{g^2 + f^2 - c}$

$r = \sqrt{(-3)^2 + (-5)^2 - 17}$

$r = \sqrt{9 + 25 - 17}$

$r = \sqrt{17}$

$C(3, 5), \ r = \sqrt{17}$

14B

2(f) $\frac{x^2}{2} + \frac{y^2}{2} + x - \frac{y}{2} - \frac{1}{2} = 0$

Not in form $x^2+y^2+2gx+2fy+c=0$ so multiply both sides by 2

$x^2 + y^2 + 2x - y - 1 = 0$

$2gx = 2x$ $\quad$ $2fy = -y$ $\quad$ $c = -1$

$g = 1$ $\quad$ $f = -\frac{1}{2}$

$C(-g, -f)$

$C(-1, \frac{1}{2})$

$r = \sqrt{g^2 + f^2 - c}$

$= \sqrt{1^2 + (-\frac{1}{2})^2 - (-1)}$

$= \sqrt{1 + \frac{1}{4} + 1}$

$= \sqrt{9/4}$

$= 3/2$

$C(-1, \frac{1}{2}),\ r = \frac{3}{2}$

Note: Answer wrong in Book answers

14B

(4) $x^2 + y^2 + 4x - 8y - 5 = 0$

$(2,k)$ lies on circumference

$(2)^2 + k^2 + 4(2) - 8k - 5 = 0$

$4 + k^2 + 8 - 8k - 5 = 0$

$k^2 - 8k + 7 = 0$

$(k-7)(k-1) = 0$

$\underline{\underline{k = 7,\ k = 1}}$

(5) $x^2 + y^2 + 4x - 8y - 5 = 0$

$(m,4)$ lies on circumference

$m^2 + (4)^2 + 4m - 8(4) - 5 = 0$

$m^2 + 16 + 4m - 32 - 5 = 0$

$m^2 + 4m - 21 = 0$

$(m+7)(m-3) = 0$

$\underline{\underline{m = -7,\ m = 3}}$

14B

(6) $x^2+y^2+6x-10y+c=0$

(9,10) lies on circumference

$$(9)^2+(10)^2+6(9)-10(10)+c=0$$

$$81+100+54-100+c=0$$

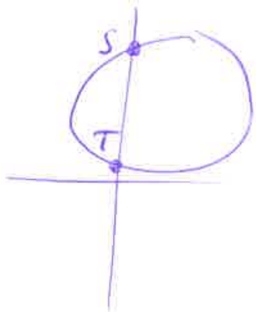
$$135+c=0$$

$$\underline{\underline{c=-135}}$$

(8) $x^2+y^2-12x-10y+24=0$

cuts y-axis at S and T

i.e. $x=0$ at S and T



$$y^2-10y+24=0$$

$$(y-6)(y-4)=0$$

$$y=6 \quad y=4$$

S(6,0) T(4,0)

$\underline{\underline{\text{line } ST = 2\text{ units}}}$

14B

(10) (a) $x^2+y^2+4x-2y-k=0$

$2gx=4x \qquad 2fy=-2y \qquad c=-k$

$g=2 \qquad f=-1$

for a circle to exist $g^2+f^2-c>0$

$(2)^2+(-1)^2--k>0$

$5+k>0$

$\underline{\underline{k>-5}}$

(10) (c) $x^2+y^2+6x-2ky+13=0$

$2gx=6x \qquad 2fy=-2ky \qquad c=13$

$g=3 \qquad f=-k$

for a circle to exist $g^2+f^2-c>0$

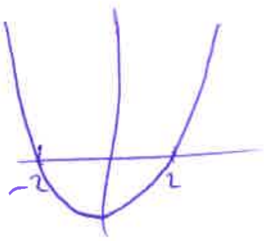
$3^2+(-k)^2-13>0$

$k^2-4>0$

$(k+2)(k-2)>0$

Solve $(k+2)(k-2)=0$ to find roots

$k=-2, k=2$



$k^2-4>0$ $\qquad$ $\underline{\underline{k<-2 \text{ and } k>2}}$

14B

(11) $x^2 + y^2 + 20x - 16y - 5 = 0$

$2g = 20 \quad 2f = -16 \quad c = -5$

$g = 10 \quad f = -8$

$C_1(-10, 8)$; $r = \sqrt{10^2 + (-8)^2 - -5}$

$r_1 = 13$

$= \sqrt{169}$

$= 13$

$x^2 + y^2 - 52x + 14y + 49 = 0$

$2g = -52 \quad 2f = 14 \quad c = 49$

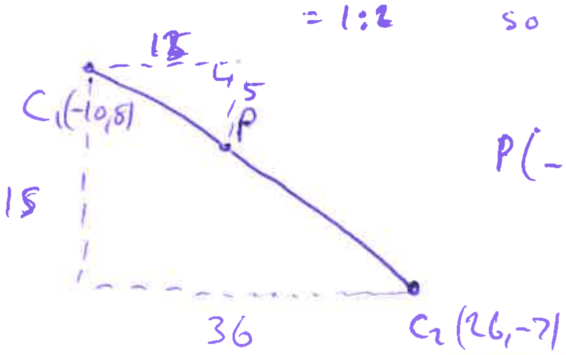
$g = -26 \quad f = 7$

$C_1(26, -7)$ $\quad r = \sqrt{(-26)^2 + (7)^2 - 49}$

$r_2 = 26$ $\quad r = 26$

$r_1 : r_2 = 13 : 26$

$= 1 : 2$ $\quad$ so $\vec{C_1P} = \frac{1}{3}\vec{C_1C_2}$



$P(-10+12, 8-5)$

$P(2, 3)$

or

	13	:	26	
	1	:	2	
C_1		P		C_2
-10	$+12$	2	$+12$ $\;14$ $\;+12$	26
8	-5	3	-5 $\;-2$ $\;-5$	-7

$P(2, 3)$

14B

12 $\quad x^2+y^2-8x-6y-11=0$

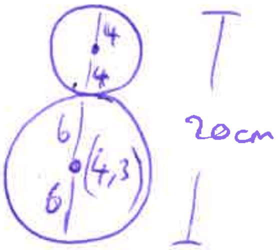
$2g=-8 \quad 2f=-6 \quad c=-11$

$g=-4 \quad f=-3$

$\underline{\underline{C(4,3)}} \qquad r=\sqrt{(-4)^2+(-3)^2-(-11)}$

$\underline{\underline{r=6}} \qquad r=\sqrt{36}$

$r=6$



Diameter body $= 2\times 6 = 12$

Diameter head $= 20-12 = 8$

radius head $= 4$

centre head 10 above centre body

$C_{head}(4,13)$

$$(x-4)+(y-13)^2=4^2$$

$$\underline{\underline{(x-4)^2+(y-13)^2=16}}$$

$\left(\text{or} \quad x^2+y^2-8x-26y+169=0\right)$

14B

(13) $x^2+y^2+10x-55=0$

$2g=10 \quad 2f=0 \quad c=-55$

$g=5 \quad f=0$

$r_1=\sqrt{(5)^2+(0)^2--55}$

$=\sqrt{80}$

$=\sqrt{16}\sqrt{5}$

$=\underline{4\sqrt{5}}$

$\underline{\underline{C_1(-5,0)}} \quad \underline{\underline{r_1=4\sqrt{5}}}$

$x^2+y^2-26x-18y+205=0$

$2g=-26 \quad 2f=-18 \quad c=205$

$g=-13 \quad f=-9$

$r_2=\sqrt{(-13)^2+(-9)^2-205}$

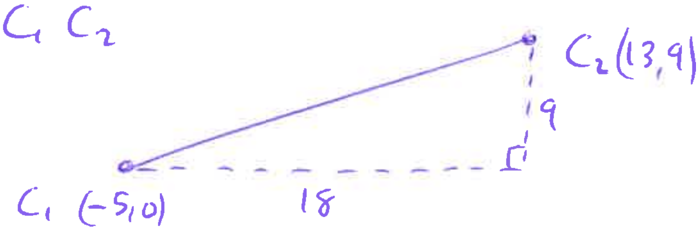
$r_2=\sqrt{45}$

$r_2=\sqrt{9}\sqrt{5}$

$r_2=3\sqrt{5}$

$\underline{C_2(13,9)} \quad \underline{\underline{r_2=3\sqrt{5}}}$

Distance C_1C_2



$C_1C_2=\sqrt{18^2+9^2}$

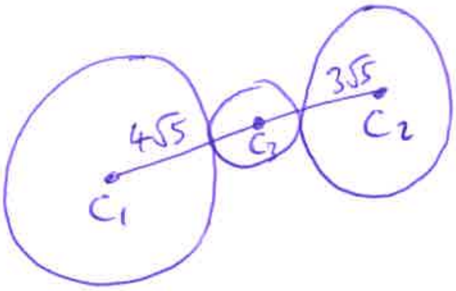
$=\sqrt{405}$

$=\sqrt{81}\sqrt{5}$

$=9\sqrt{5}$

14B

(13) continued



$C_1C_2 = 9\sqrt5$

$$\text{Diameter } C_3 = 9\sqrt5 - 4\sqrt5 - 3\sqrt5$$
$$= 2\sqrt5$$

$$r_3 = \sqrt5$$

$5\sqrt5 : 4\sqrt5$

$5 : 4$

$C_1 \quad C_3 \quad C_2$

$\begin{pmatrix}-5\\0\end{pmatrix}$ -3 -1 1 3 $\begin{pmatrix}5\\5\end{pmatrix}$ 7 9 11 $\begin{pmatrix}13\\9\end{pmatrix}$

$\phantom{\begin{pmatrix}-5\\0\end{pmatrix}}$ 1 2 3 4 $\quad$ 6 7 8

or $\overrightarrow{C_1C_3} = \frac{5}{9}(C_1C_2)$

$$C_3(5,5) \quad r_3 = \sqrt5$$

$$(x-5)^2 + (y-5)^2 = (\sqrt5)^2$$
$$(x-5)^2 + (y-5)^2 = 5$$

or $x^2 + y^2 - 10x - 10y + 45 = 0$

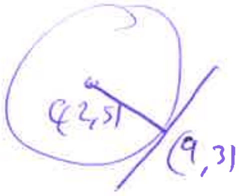
14C

1) $x^2+y^2-4x-10y-5=0$

$2g=-4 \qquad 2f=-10$

$g=-2 \qquad f=-5$

$C(2,5)$



$M_{rad} = \frac{3-5}{9-2}$

$= \frac{-2}{7}$

$M_{tan} = \frac{7}{2}$ as radius perpendicular to tangent.

$y-b=m(x-a)$

$y-3=\frac{7}{2}(x-9)$ use $(9,3)$ as it lies on tangent.

$2(y-3)=7(x-9)$

$2y-6=7x-63$

$\underline{\underline{2y=7x-57}}$

or $2y-7x=-57$

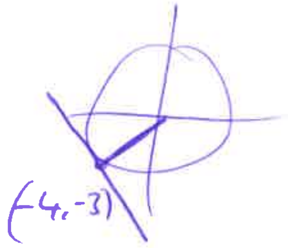
or $2y-7x+57=0$

or $y=\frac{7}{2}x-\frac{57}{2}$

14C

(3) $x^2+y^2=25$

$C(0,0)$



$M_{rad} = \dfrac{0--3}{0--4}$

$M_{rad} = 3/4$

$M_{tan} = -\dfrac{4}{3}$ as radius perp to tangent.

$y-b = m(x-a)$

$y--3 = -\dfrac{4}{3}(x--4)$

$y+3 = -\dfrac{4}{3}(x+4)$

$3(y+3) = -4(x+4)$

$3y+9 = -4x-16$

$3y+4x+25=0$

or $3y+4x=-25$

or $3y=-4x-25$

or $y=-\dfrac{4}{3}x-\dfrac{25}{3}$

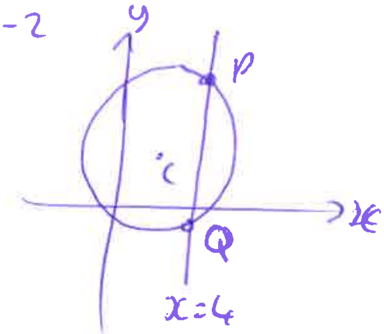
14C

4 $x^2+y^2-2x-4y-20=0$

$2g=-2 \quad 2f=-4$

$g=-1 \quad f=-2$

$C(1,2)$



at P and Q $x=4$

$(4)^2+y^2-2(4)-4y-20=0$

$16+y^2-8-4y-20=0$

$y^2-4y-12=0$

$(y-6)(y+2)=0$

$y_P=6 \quad y_Q=-2$

$P(4,6)$, $Q(-2,6)$

$M_{cp} = \frac{6-2}{4-1}$

$= \frac{4}{3}$

$M_{tan} = -3/4$

$y-b=m(x-a)$

$y-6=-\frac{3}{4}(x-4)$

14C

(4) continued

$$4(y-6) = -3(x-4)$$

$$4y - 24 = -3x + 12$$

$$4y + 3x - 24 = 12$$

$$4y + 3x = 36$$

(5) $2y + x = 31$

$$x = 31 - 2y$$

$$x^2 + y^2 - 6x - 8y - 55 = 0$$

$$(31-2y)^2 + y^2 - 6(31-2y) - 8y - 55 = 0$$

$$(31-2y)(31-2y) + y^2 - 186 + 12y - 8y - 55 = 0$$

$$961 - 124y + 4y^2 + y^2 - 186 + 12y - 8y - 55 = 0$$

$$5y^2 - 120y + 720 = 0$$

$$5(y^2 - 24y + 144) = 0$$

$$5(y-12)(y-12) = 0$$

$$5(y-12)^2 = 0$$

repeated root $\therefore$ line is a tangent to the circle

14C

5 continued

$$y - 12 = 0$$
$$y = 12$$

$$2y + x = 31$$

$$2(12) + x = 31$$
$$24 + x = 31$$
$$x = 7$$

$\underline{\underline{(7, 12)}}$ is point of contact

14C

6 $3y - 5x = 11$

$$3y = 5x + 11$$

$$y = \frac{5x+11}{3}$$

$$x^2 + y^2 - 14x - 8y + 31 = 0$$

$$x^2 + \left(\frac{5x+11}{3}\right)^2 - 14x - 8\left(\frac{5x+11}{3}\right) + 31 = 0$$

$$x^2 + \left(\frac{5x+11}{3}\right)\left(\frac{5x+11}{3}\right) - 14x - \frac{40x}{3} - \frac{88}{3} + 31 = 0$$

$$x^2 + \frac{25x^2 + 110x + 121}{9} - 14x - \frac{40x}{3} - \frac{88}{3} + 31 = 0$$

multiply both side of equation by 9 to eliminate fractions

$$9x^2 + 25x^2 + 110x + 121 - 126x - 120x - 264 + 279 = 0$$

$$34x^2 - 136x + 136 = 0$$

$$b^2 - 4ac$$

$$= (-136)^2 - 4(34)(136)$$

$= 0$ ∴ one point of contact, the line is a tangent to the circle.

14C

(10) $x^2 + y^2 = 5$, $y = x + k$

$x^2 + (x+k)^2 = 5$

$x^2 + x^2 + 2kx + k^2 = 5$

$2x^2 + 2kx + k^2 - 5 = 0$

$a = 2 \quad b = 2k \quad c = k^2 - 5$

$b^2 - 4ac = 0$ if line is a tangent to the curve

$(2k)^2 - 4(2)(k^2-5) = 0$

$4k^2 - 8(k^2-5) = 0$

$4k^2 - 8k^2 + 40 = 0$

$-4k^2 + 40 = 0$

$-4(k^2-10) = 0$

$-4(k+\sqrt{10})(k-\sqrt{10}) = 0$

$k = -\sqrt{10}, \ k = \sqrt{10}$

14C

(11) $(x-1)^2 + (x-k)^2 = 2$

$x^2 - 2x + 1 + x^2 - 2kx + k^2 = 2$

$2x^2 - 2x - 2kx + k^2 + 1 - 2 = 0$

$2x^2 + x(-2-2k) + k^2 - 1 = 0$

$a = 2 \quad b = -2-2k \quad c = k^2 - 1$

$b^2 - 4ac = 0$ for a tangent.

$(-2-2k)^2 - 4(2)(k^2-1) = 0$

$4 + 8k + 4k^2 - 8k^2 + 8 = 0$

$-4k^2 + 8k + 12 = 0$

$-4(k^2 - 2k - 3) = 0$

$-4(k-3)(k+1) = 0$

$k = -1, \; k = 3$

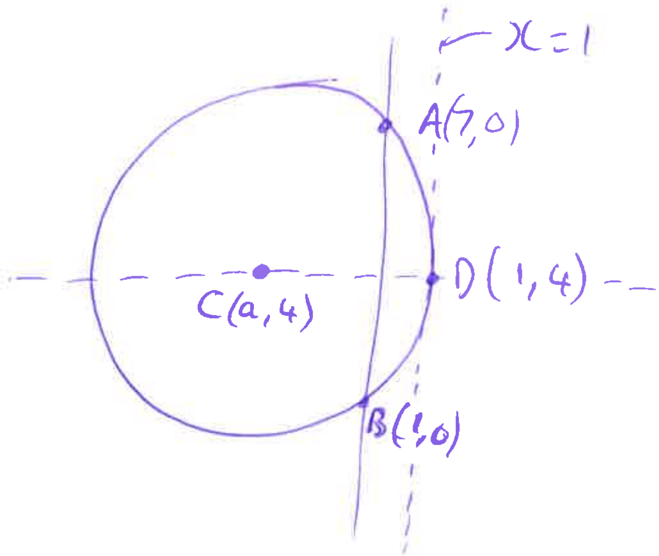
14C

12

Use symmetry of circle

$C(a,4)$

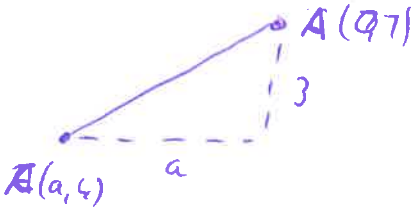
$D(1,4)$ ($x=1$ is a tangent)



consider radius AC and AD

$AC: \quad a^2+3^2 = r^2$

$a^2+9 = r^2$



$CD: \quad (1-a)^2+0^2 = r^2$

$r_{AC} = r_{CD}$

$$a^2+9 = (1-a)^2$$

$$a^2+9 = 1-2a+a^2$$

$$9 = 1-2a$$

$$2a = -8$$

$$a = -4$$

$C(-4,4)$

$$r^2 = a^2+3^2$$

$$r^2 = (-4)^2+3^2$$

$$r^2 = 25$$

$$\underline{(x+4)^2+(y-4)^2 = 25}$$

14C

(14) $C_1(-5,2)$ $\quad r_1 = \sqrt{(-5)^2+(2)^2-(-11)}$

$r_1 = \sqrt{40}$

$r_1 = 2\sqrt{10}$

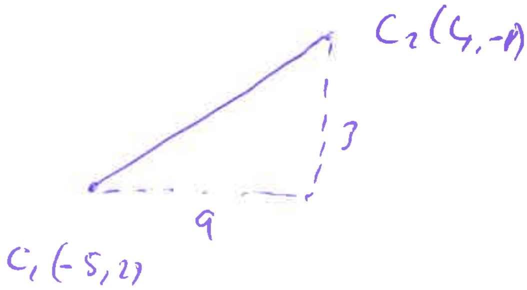
$C_2(4,-1)$ $\quad r_2 = \sqrt{4^2+(-1)^2-7}$

$r_2 = \sqrt{10}$

distance $C_1C_2 = \sqrt{9^2+3^2}$

$= \sqrt{90}$

$= 3\sqrt{10}$



ratio $r_1 : r_2 = 2\sqrt{10} : \sqrt{10} = 2:1$

$$\begin{array}{cccc} C_1 & & P & C_2 \\ -5 & -2 & \begin{pmatrix} 1 \\ 0 \end{pmatrix} & 4 \\ 2 & 1 & & -1 \end{array}$$

$P(1,0)$

Note: as distance $C_1C_2 = r_1 + r_2$ the circles touch

14D

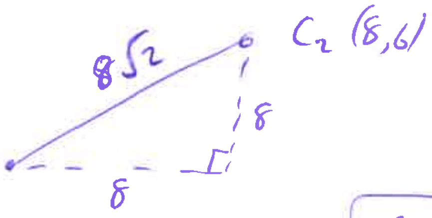
1 (a) $C_1(0,-2)$ $\qquad r_1 = \sqrt{0^2+(-2)^2-(-32)}$

$r_1 = 6$

$C_2(8,6)$ $\qquad r_2 = \sqrt{(8)^2+(6)^2-75}$

$r_2 = 5$

$C_1C_2 = 8\sqrt{2}$



$r = \sqrt{8^2+8^2}$

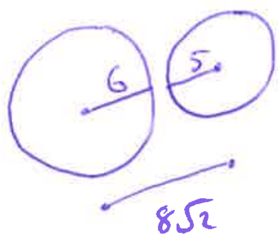
$r = \sqrt{128}$

$r = 8\sqrt{2}$

$C_1C_2 = 8\sqrt{2}$
$= \sqrt{128}$

$r_1+r_2 = 6+5 =$
$= 11$
$= \sqrt{121}$

Distance $C_1C_2 > r_1+r_2$ $\therefore$ circles do not intersect



14 i)

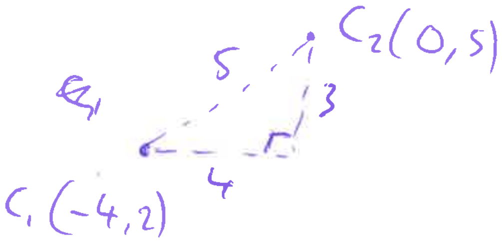
1(c) $C_1(-4, 2)$ $\qquad r_1 = \sqrt{(-4)^2 + (2)^2 - 11}$

$r_1 = 3$

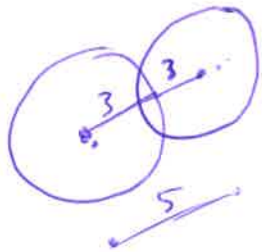
$C_2(0, 5)$ $\qquad r_2 = \sqrt{(0)^2 + (5)^2 - 16}$

$r_2 = 3$

$C_1C_2 = 5$



$C_1C_2 < r_1 + r_2 \quad \therefore$ circles intersect at 2 points



Note: For circles to just touch they would need to be 6 $(r_1 + r_2 = 6)$ apart, as they are 5 apart they ~~cross~~ intersect at 2 points.

14D

1(e) $C_1 = (-8, -4)$

$r_1 = \sqrt{(-8)^2 + (-4)^2 - (-320)}$

$= \sqrt{400}$

$r_1 = 20$

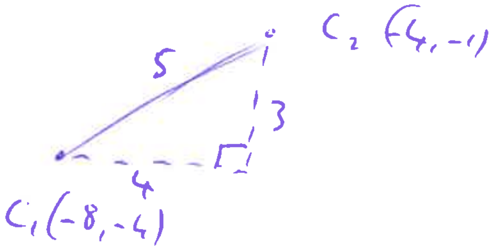
$C_2 = (-4, -1)$

$r_2 = \sqrt{(-4)^2 + (-1)^2 - (-47)}$

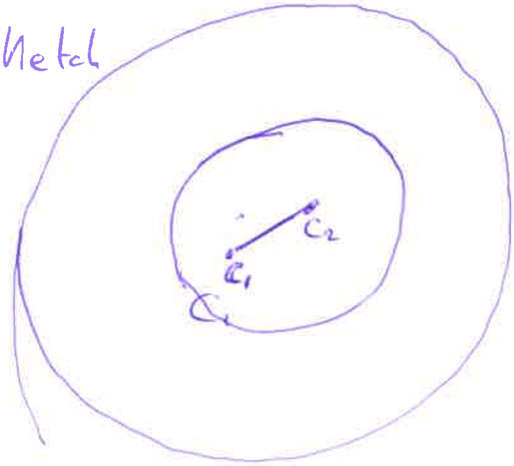
$r_2 = \sqrt{64}$

$r_2 = 8$

$C_1C_2 = 5$



Quick sketch



$r_1 = 20$

$r_2 = 8$

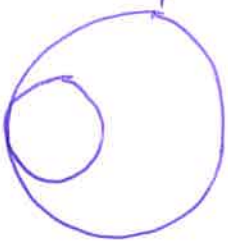
$C_1C_2 = 5$

Circles do not intersect as $C_1C_2 < r_1 - r_2$, circle C_2 is contained within C_1.

14D

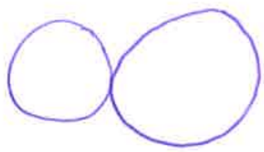
(2) Question does not specify if circles touch internally or externally.

internally



$d = r_1 - r_2$

externally



$d = r_1 + r_2$

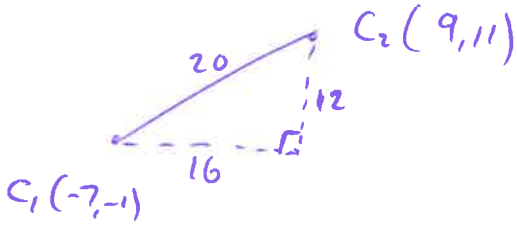
(a) $C_1(9,11)$ $\quad r_1 = \sqrt{9^2 + 11^2 - 23}$

$r_1 = 15$

$C_2(-7,-1)$ $\quad r_2 = \sqrt{(-7)^2 + (-1)^2 + p}$

$r_2 = \sqrt{50 + p}$

$C_1C_2 = 20$



If touch internally

$C_1C_2 = r_2 - r_1$ (as $r_2 > r_1$ for this to occur)

$20 = r_2 - 15$

$r_2 = 35$

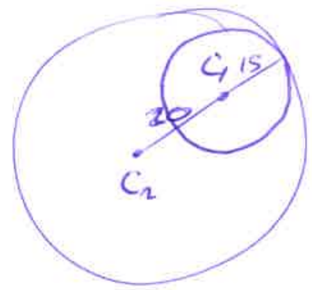
$\sqrt{50 + p} = 35$

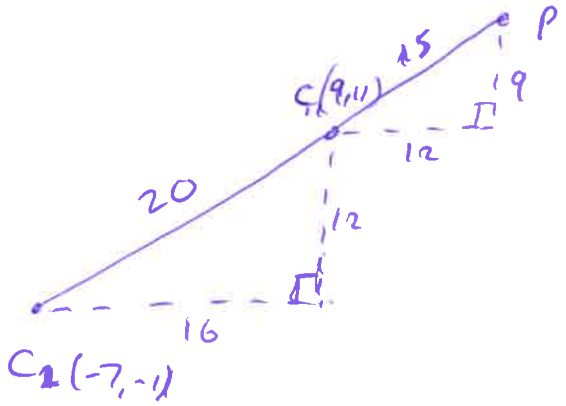
$50 + p = 1225$

$p = -1175$

14D

2(b) If touch internally





$C_1P = \frac{3}{4} C_2C_1$

$P(9+12, 11+9)$

$\underline{\underline{P(21, 20)}}$

2(a) If circles touch externally

$C_1C_2 = r_1 + r_2$

$20 = 15 + r_2$

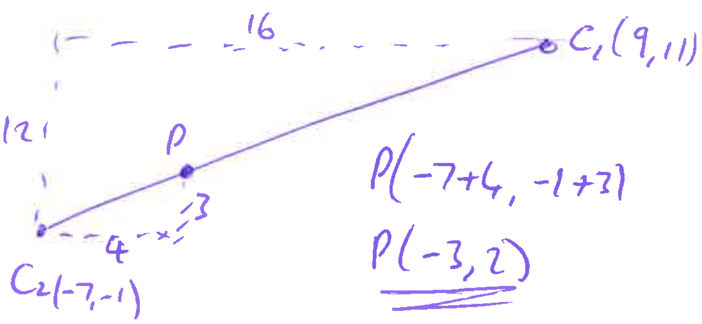
$\underline{\underline{r_2 = 5}}$

$\sqrt{50 - p} = 5$

$50 - p = 25$

$\underline{\underline{p = 25}}$

(b) $C_2P = \frac{1}{4} C_1C_2$



$P(-7+4, -1+3)$

$\underline{\underline{P(-3, 2)}}$